3.5 Hamiltonian systems with damping

Exercise 3.5.1. *Consider the potential function $V(x) = -\frac{1}{2}x^2$ for a particle with mass $m = 4$. The corresponding damped Hamiltonian system, with damping constant $b = 3$, is*

$$\frac{dx}{dt} = v \qquad 4\frac{dv}{dt} = x - 3v.$$

Analyze this system via the following.

1. *Use eigenstuff to find the general solution.*
2. *Use eigenstuff to sketch the phase portrait.*
3. *Consider the solution with $x(0) = -1$ and $v(0) = 1$. Draw the trajectory of this solution in the phase portrait. Then draw the trajectory of this solution in the energy diagram.*
4. *Consider a solution with $x(0) = -1$ and $v(0) > 1$. Draw the trajectory in both the phase portrait and energy diagram.*
5. *Consider a solution with $x(0) = -1$ and $v(0) < 1$. Draw the trajectory in both the phase portrait and energy diagram.*

What can you conclude generally about solutions to this system?

Exercise 3.5.2. *The equation*

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x^2 = 3$$

can be perceived as a model for some non-linear oscillation with friction (damping).

1. *Write down the corresponding frictionless (undamped) system. What is the conserved energy for that system?*
2. *Show that the energy for the damped system is monotone decreasing.*
3. *Draw the energy diagram for the damped system.*
4. *Then discuss the possibilities for the long-term fate of the solutions – what are the possible types of solutions?*