Chapter 4

Oscillations and resonance

4.1 Modeling oscillations

Exercise 4.1.1. *Consider a generic oscillator equation*

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} = ky = 0.$$

Suppose that $y_1(t)$ and $y_2(t)$ are solutions. Explain why it follows $\alpha y_1(t) + \beta y_2(t)$ is a solution for any choice of constants α and β. This is called the ***superposition principle****.*

Exercise 4.1.2. *Consider the second order differential equation*

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0.$$

1. *Find the general solution to this differential equation.*
2. *Solve the initial value problem*

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0, \qquad y(0) = 0, \quad y'(0) = 1.$$

Exercise 4.1.3. *Find the general solution of the following equations:*

1. $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$*;*
2. $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = 0$*;*
3. $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} = 0$*;*

4. $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 0$;

5. $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$.

Exercise 4.1.4. *Consider the differential equation*

$$t^2\frac{d^2y}{dt^2} - 3t\frac{dy}{dt} + 3y = 0.$$

1. *Explain why the superposition principle applies to this equation. That is, show that that if $y_1(t)$ and $y_2(t)$ are solutions then so is $\alpha y_1(t) + \beta y_2(t)$ for any constants α and β.*

2. *Find those values of α for which the function $y(t) = t^\alpha$ solves the differential equation.*

3. *Use the superposition principle to solve the IVP:*

$$t^2\frac{d^2y}{dt^2} - 3t\frac{dy}{dt} + 3y = 0, \qquad y(1) = 2, \quad y'(1) = 4.$$