Handout 6

Mathematical approximations

6.1 Terminal tip: `ssh`

Previously we saw how to use `sftp` to `get` files from another computer. Today we show how to operate another computer remotely using ssh.

In order to connect to machine `simpson99` we use the command

```
$ ssh student@simpson99.lclark.edu - X
```

and then enter the password. The option `-X` allows the Atom window to open on the local machine.

From here you can browse around, open files in atom, etc. To close the connection type

```
$ exit
```

6.2 Approximating e

The Taylor approximation for e is

$$e = \underbrace{1}_{\text{term } 0} + \underbrace{\frac{1}{1!}}_{\text{term } 1} + \underbrace{\frac{1}{2!}}_{\text{term } 2} + \underbrace{\frac{1}{3!}}_{\text{term } 3} + \underbrace{\frac{1}{4!}}_{\text{term } 4} + \dots.$$

Note that

$$(\text{term } k) = \frac{1}{k}(\text{term } k-1).$$

In our loop we will count by term. For each k we

- add k^{th} term to approximtion
- increase counter k
- construct the next term

Try writing the loop on paper before turning to the next page.

```c
/*
program to approximate e using Taylor series
*/
#include <stdio.h>
#include <math.h>
#include <stdlib.h>

int main()
{
  // obtain order of approximation
  double order;
  printf("Enter desired order of approximation \n");
  scanf("%lf", &order);

  // construct approximation
  double approx=0; // approximation
  double k =0; // counter
  double term = 1; // term in the sum
  while (k<=order)
  {
    // add kth term to approx
    approx = approx + term;
    // increase counter k
    k++;
    // update the term
    term = (1/k)*term;
  }

  printf("The approximation of e is %lf \n", approx);
}
```

6.3 Homework: `approx-root2.c`

The Taylor approximation of $\sqrt{2}$ is given by

$$\sqrt{2} = \frac{1}{1} + \frac{1}{2} - \frac{1}{2 \cdot 4} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} + \dots$$

Write a program `approx-root2.c` that constructs approximations of $\sqrt{2}$.

6.4 More practice problems

- The Taylor approximation

$$(1+x)^{1/3} = 1 + \frac{1}{3}x - \frac{2}{3 \cdot 6}x^2 + \frac{2 \cdot 5}{3 \cdot 6 \cdot 9}x^3 - \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9 \cdot 12}x^4 + \dots$$

is valid when $-1 < x \leq 1$. Write a program that takes input x and an integer, and returns the approximation of that order. (If x is not in the valid range, your program should refuse to compute the approximation!)

- The Taylor approximation

$$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

is valid for all real x. Write a program that takes input x and an integer, and returns the approximation of that order. How does your approximation compare to the built-in cosine function?

- What does the following sum converge towards?

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \dots$$

6.5 Lab: `viete1.c`

Viète's formula for π is

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \dots$$

See https://en.wikipedia.org/wiki/Vi%C3%A8te%27s_formula for more information.

Write a program `viete1.c` that takes an integer as input and the computes an approximation of π using that many terms in Viète's formula.

6.6 Challenge: Continued fractions

Use the following continued fraction expression in order to approximate $\sqrt{2}$:

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}$$