

Exercises for Day 20

First-eq-pt-analysis

EXERCISE 20.1. Perform the equilibrium point analysis of the following predator-prey model:

$$\frac{dx}{dt} = x - xy \quad \frac{dy}{dt} = -2y + 2xy$$

That is, execute the following steps.

- Find all equilibrium solutions of the system.
- Linearize the system near each equilibrium.
- Understand the linearized models using eigenstuff.
- As much as possible, piece the phase portraits of the linearized systems together to get an approximate phase portrait of the full system.
- Find the nullclines of the system. Use a combination of the nullclines and the equilibrium point analysis in order to obtain as complete an understanding of how solutions behave as you can.
- Use **Sage** to construct a plot of the vector field for the system. Draw the nullclines and equilibrium points on your plot using a pencil.
- Write a couple sentences describing how “typical” solutions behave.

AnotherPredPrey

EXERCISE 20.2. Repeat Exercise 20.1 for the predator-prey model

$$\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{100}\right) - 0.06PR \quad \frac{dR}{dt} = -0.4R + 0.01PR.$$

YetAnotherPredPrey

EXERCISE 20.3. Repeat Exercise 20.1 for the predator-prey model:

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{100}\right) - 0.005xy \quad \frac{dy}{dt} = \frac{y}{2}\left(1 - \frac{y}{200}\right) + 0.01xy.$$

EQ-Last

EXERCISE 20.4. Consider the non-linear system

$$\frac{dx}{dt} = y \quad \frac{dy}{dt} = -x + (1 - x^2 - y^2)y.$$

- (1) There is one equilibrium solution of this system – find it!
- (2) Linearize the system near this equilibrium, and draw the phase portrait of the linearized system.
- (3) Make an educated guess about the phase portrait of the non-linear system. For your own benefit do this without any help of “technology”.
- (4) Show that $x(t) = \sin(t)$, $y(t) = \cos(t)$ is a solution of the non-linear system. What is the phase diagram of this solution?

- (5) *Now make an educated guess about the phase portrait of the non-linear system. (Remember: phase curves for nice systems do not intersect!!!)*
- (6) *Construct a phase plot using **Sage**.*
- (7) *Comment on what you learned about linearization from this problem.*