DAY 10

Exam

Here are some problems you can use to prepare for the exam.

EXERCISE 10.1. *Solve the initial value problem*

$$\frac{dy}{dt} = y^3 \qquad y(0) = 2.$$

EXERCISE 10.2. *Solve the initial value problem*

$$\frac{dy}{dt} = t\, y^2 \qquad y(0) = 1.$$

EXERCISE 10.3. *Consider a* 200 L *tank, which is initially full of salt water. The water in the tank is being drained at a rate of* 15 $^{\text{L}}/_{\text{min}}$. *At the same time, a solution with concentration* 12 $^{\text{g}}/_{\text{L}}$ *is entering the tank at a rate of* 10 $^{\text{L}}/_{\text{min}}$. *During this process, the tank is being perfectly mixed.*

(1) *Write down a differential equation which describes how the amount of salt in the tank changes in time.*
(2) *Find the propagator function associated to your differential equation.*
(3) *Suppose that initially there are* 100 g *of salt in the tank. Find an expression for the amount of salt in the tank at time* π. *You do not need to simply your expression.*

EXERCISE 10.4. *Consider a* 100 L *tank, which is initially only half full of salt water. The water in the tank is being drained at a rate of* 5 $^{\text{L}}/_{\text{min}}$. *At the same time, a solution with concentration* 12 $^{\text{g}}/_{\text{L}}$ *is entering the tank at a rate of* 10 $^{\text{L}}/_{\text{min}}$. *During this process, the tank is being perfectly mixed.*

(1) *Write down a differential equation which describes how the amount of salt in the tank changes in time. For what time*

period is your differential equation a valid description of the situation described above?

(2) *Find the propagator function associated to your differential equation.*

(3) *Suppose that initially there are* 100 g *of salt in the tank. Find an expression for the amount of salt in the tank at time* π*. You do not need to simply your expression (but you do need to complete the integration).*

EXERCISE 10.5. *Consider the differential equation*

$$\frac{dP}{dt} = P^3 - 4P^2 + 4P.$$

(1) *Make a sketch of the slope field for differential equation.*

(2) *Determine the stability of each of the equilibrium points.*

(3) *Based on your slope field, describe the long term behavior of the solution to the equation having initial condition* $P(0) = 1$*.*

EXERCISE 10.6. *Consider the differential equation*

$$\frac{dP}{dt} = P^3 - P^2 - 12P.$$

(1) *Make a sketch of the slope field for differential equation.*

(2) *Determine the stability of each of the equilibrium points. Explain your reasoning.*

(3) *Based on your slope field, describe the long term behavior of the solution to the initial value problem with initial condition* $P(0) = 1$*.*