MATH 131: WORKSHEET 10

Cumulative effect

1. Computing cumulative effect

As in class yesterday we are assuming that a population of bees is growing according to the rate

$$r(t) = 7t \quad \text{bees / day.}$$

Our plan is to compute the cumulative effect of this growth from time $t = 0$ days to time $t = 10$ days.

(1) The plan is to measure the growth rate at n different times. Explain why the time gap between measurements is

$$\Delta t = \frac{10}{n}$$

and that the times where we are measuring the rate are

$$t_1 = \frac{10}{n}, \quad t_2 = 2\frac{10}{n}, \quad t_3 = 3\frac{10}{n}, \quad \dots \quad t_n = n\frac{10}{n}.$$

(2) What is the growth rate at each of these times? That is, compute

$$r(t_1) = \_\_\_\_, \quad r(t_2) = \_\_\_\_, \quad r(t_3) = \_\_\_\_, \quad \dots \quad r(t_n) = \_\_\_\_.$$

(3) We are going to use the symbol S_n to represent the cumulative effect computed using the n measurements Explain why

$$S_n = r(t_1)\Delta t + r(t_2)\Delta t + r(t_3)\Delta t + \cdots + r(t_n)\Delta t.$$

Show that

$$S_n = 7\left(\frac{10}{n}\right)^2 (1 + 2 + 3 + \cdots + n).$$

(4) Use the trick from class yesterday to show that

$$S_n = 7\left(\frac{10}{n}\right)^2 \left(\frac{n(n+1)}{2}\right)$$

(5) We now want to take the limit as the number of measurements gets large. Show that

$$\lim_{n\to\infty} S_n = 350.$$

2. An alternate perspective

Here we explore the cumulative effect problem from a different perspective.

(1) Let $P(t)$ be the function that tells us the population of bees at time t. At this point, we don't have a formula for $P(t)$. Nevertheless, explain why the cumulative effect of the growth from $t = 0$ to $t = 10$ should be equal to

$$P(10) - P(0).$$

(2) We are assuming that the rate of change of the population of bees is $7t$. Explain why this assumption means that $P'(t) = 7t$.

(3) Explain how to deduce that

$$P(t) = \frac{7}{2}t^2 + (\text{some number}).$$

(4) Compute $P(10) - P(0)$. What happens to the term "some number"? Should this number matter?

(5) Compare your answer to the cumulative effect we computed in the previous section. Make an intelligent remark.

3. More work

(1) Repeat parts 1 and 2 of this worksheet where the growth rate is $r(t) = 6t^2$. You will need the fact that

$$1^1 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(2) In general, it seems that there are two different ways to compute cumulative effect: adding lots of terms and subtracting. Try to formulate a general statement that relates these two concepts/approaches.