MATH 131: WORKSHEET 8

Optimization problems

(1) (Famous Fence Problem 1) Suppose we have 100 meters of fencing material, and that we use this material to surround a rectangular region.

  (a) Let x be the width of the region. Given that we have only 100 meters of material, what must the length of the region be? Your answer will be in terms of x.

  (b) What is the area of the region? Write your answer in the form $A =$ ____, where the blank line is a formula in terms of x.

  (c) We can view A as a function of x. Compute $\frac{dA}{dx}$ and find the critical point of A. Show that the critical point is a local maximum.

  (d) You have just discovered what the width of the region should be in order to maximize area! Explain why your answer makes "intuitive sense." What is the area of this largest region?

(2) (Famous Fence Problem 1) Suppose again that we have 100 meters of fencing, and again that we are going to use this material to fence in a rectangular region. However, this time, we using a pre-existing wall for one side of the region. . . thus the 100 meters of fencing only needs to be used for three sides of the region.

What are the dimensions of the region with the most area? What is this maximum area?

(3) (Famous Soup Can Problem)

  (a) Suppose we have a cylindrical soup can with height h and radius r. What is the surface area of the can (including top, bottom, and sides)? What is the volume enclosed by the can?

(b) Suppose that we want to construct a can that has a volume of 100 cubic centimeters. What relationship between h and r does this require?

(c) Use the relationship from the previous part in order to write the surface area A as a function of only one variable (either h or r, your choice).

(d) Find the minimum of your area function. What are the dimensions of the can holding 100 cubic centimeters and made from the least amount of material?

(4) Suppose that we modify the previous problem so that we are seeking to make a cylindrical cup that holds 100 cubic centimeters. The cup has sides and a base, but no top. What are the dimensions of the cup that uses the least material to make?

(5) Suppose we make a rectangular box, with a square base and no lid, that holds 500 cubic centimeters. What are the dimensions of the box that uses the least amount of material?

(6) Suppose that we make a rectangular box, having a square base and including flaps that fold to make a lid, that holds 500 cubic centimeters. What are the dimensions of the box that uses the least amount of material? Draw a picture.