MATH 131: WORKSHEET 7

Derivatives of trigonometric functions

(1) Complete the following trig identity: $\sin(x+h) = \_\_\_\_$. Use the identity to show that

$$\frac{\sin(x+h) - \sin(x)}{h} = \sin(x)\,\frac{\cos(h) - 1}{h} + \cos(x)\,\frac{\sin(h)}{h}.$$

(2) Based on examining graphs, what are the following limits?

$$\lim_{h\to 0}\left[\frac{\cos(h)-1}{h}\right] \quad \text{and} \quad \lim_{h\to 0}\left[\frac{\sin(h)}{h}\right]$$

Use these limits to compute the derivative of the sine function.

(3) Recall that $(\sin(x))^2 + (\cos(x))^2 = 1$. Use this, together with the chain rule, to see that

$$2\sin(x)\,\frac{d}{dx}[\sin(x)] + 2\cos(x)\,\frac{d}{dx}[\cos(x)] = 0.$$

Use your knowledge of the derivative of the sine function to figure out what is the derivative of the cosine function.

(4) Compute the following:

(a) $\frac{d}{dx}\left[4x^2 - \cos(x)\right]$

(b) $\frac{d}{dx}\left[4x^2\cos(x)\right]$

(c) $\frac{d}{dx}[\sin(3x)]$

(d) $\frac{d}{dx}\left[\cos(4x^2 - 5)\right]$

(e) $\frac{d}{dx}[x\sin(2x)]$

(f) $\frac{d}{dx}\left[\frac{\cos(x)}{x}\right]$

(5) Compute, and simplify, the derivatives of the functions

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \sec(x) = \frac{1}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \qquad \csc(x) = \frac{1}{\sin(x)}.$$