MATH 131: WORKSHEET 6

The product and chain rule

The purpose of this worksheet is to practice the product and chain rules:

Product rule:

$$\frac{d}{dx}\,[f\,g] = \frac{d}{dx}[f]\,g + f\,\frac{d}{dx}[g]$$

Chain rule:

$$\frac{d}{dx}[f(g)] = f'(g)\,\frac{d}{dx}[g]$$

I also want you to practice finding critical points and increasing/decreasing regions.

(1) Compute $\frac{d}{dx}[\sqrt{1+x^2}]$ and $\frac{d}{dx}[x^3\,\sqrt{1+x^2}]$.

(2) Consider the function $f(x) = \dfrac{1}{9+x^2}$
- Explain why the function f does not have any roots.
- Write the function f in a form that does not involve fractions.
- Compute $f'(x)$
- Find the critical points of f and determine on which regions f is increasing/decreasing. Use these to identify the local min/max points of f.
- Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
- Use the information you have gathered to sketch the graph of f.

(3) Consider the function $f(x) = \dfrac{x}{9+x^2}$
- Find the roots of f.
- Write the function f in a form that does not involve fractions.
- Compute $f'(x)$
- Find the critical points of f and determine on which regions f is increasing/decreasing. Use these to identify the local min/max points of f.
- Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
- Use the information you have gathered to sketch the graph of f.

(4) (Challenge) Suppose we have a function that takes the form

$$f(x) = \frac{\heartsuit}{\clubsuit},$$

where $\heartsuit$ and $\clubsuit$ are some functions involving x.

- Explain how to write $f(x)$ in a form that does not involve any fractions.
- Use the form of f that does not involve a fraction to compute the derivative of $f(x)$. Your derivative will involve terms of the form $\frac{d}{dx}[\heartsuit]$ and $\frac{d}{dx}[\clubsuit]$.
- Re-arrange your formula for $f'(x)$ to obtain

$$f'(x) = \frac{\frac{d}{dx}[\heartsuit]}{\clubsuit} - \frac{\heartsuit \frac{d}{dx}[\clubsuit]}{(\clubsuit)^2}$$

Then get a common denominator in order to conclude that

$$\boxed{\frac{d}{dx}\left[\frac{\heartsuit}{\clubsuit}\right] = \frac{\frac{d}{dx}[\heartsuit]\clubsuit - \heartsuit \frac{d}{dx}[\clubsuit]}{(\clubsuit)^2}}$$

This formula is called the **quotient rule**.

- Use the quotient rule to compute

$$\frac{d}{dx}\left[\frac{x}{9+x^2}\right]$$

How does your answer compare to your previous answer?

- Use the quotient rule to compute

$$\frac{d}{dx}\left[\frac{1}{9+x^2}\right]$$

How does your answer compare to your previous answer?

- Use the quotient rule to compute

$$\frac{d}{dx}\left[\frac{x^3}{\sqrt{9+x^2}}\right]$$