MATH 131: WORKSHEET 5

Calculus and the graphs of functions

The goal of this worksheet is for you to learn more about how to graphically interpret the derivative of a function. **You only need to turn in Problem (4).** The other problems should be discussed with your classmates.

(1) We begin with some definitions.
- A function f is said to be **increasing** at input x if $f'(x) > 0$.
- A function f is said to be **decreasing** at input x if $f'(x) < 0$.

Given that the derivative $f'(x)$ describes the rate of change, why is are these reasonable definitions?

(2) Consider the function $f(x) = 4x - x^2$. For what values of x is f increasing? For what values of x is f decreasing? Draw a sketch of f. Do your computations agree with your sketch?

(3) We make some more definitions:
- A function f has a **local maximum** point at x_* if $f(x_*)$ is larger than $f(x)$ for all x close to x_*.
- A function f has a **local minimum** point at x_* if $f(x_*)$ is less than $f(x)$ for all x close to x_*.

(a) Suppose you know where a function is increasing and where it is decreasing. How can you find the minimum and maximum points?

(b) What must the derivative of a function be at local minimum and maximum points?

(4) Consider the function $f(x) = x^3 - 6x^2 + 9x - 54$.
- For what values of x is f increasing?
- For what values of x is f decreasing?
- Find the local minimum of f.
- Find the local maximum of f.