MATH 131: WORKSHEET 4

Basic derivatives

(1) Compute the following derivatives using limits.

(a) $\frac{d}{dx}[x]$ (b) $\frac{d}{dx}\left[x^2\right]$ (c) $\frac{d}{dx}\left[x^3\right]$

(2) In this problem we deduce a formula for $\frac{d}{dx}[x^p]$, where p is some positive integer.

(a) As a first step, show that

$$(x+h)^4 = x^4 + h4x^3 + h^2(\text{stuff with } x \text{ and } h).$$

Use this to deduce that

$$\lim_{h\to 0}\left[\frac{(x+h)^4 - x^4}{h}\right] = \lim_{h\to 0}\left[4x^3 + h(\text{stuff with } x \text{ and } h)\right] = 4x^3.$$

(b) Now show that

$$(x+h)^p = x^p + hpx^{p-1} + h^2(\text{stuff with } x \text{ and } h).$$

Use this to deduce that $\frac{d}{dx}[x^p] = px^{p-1}$.

(c) Use the shortcut rule you just developed in order to compute $\frac{d}{dx}\left[x^{25}\right]$.

(3) The argument that you used to show that $\frac{d}{dx}[x^p] = px^{p-1}$ does not work when the power $p = 1/2$. Nevertheless, it turns out that the formula does work for all powers. Verify this by showing that

$$\lim_{h\to 0}\left[\frac{\sqrt{x+h}-\sqrt{x}}{h}\right] = \frac{1}{2\sqrt{x}}.$$

(4) Compute $\frac{d}{dx}\left[\frac{2}{\sqrt[5]{x}}\right]$, $\frac{d}{dx}[4x^7 - 23x^2 + 5\sqrt{x}]$, and $\frac{d}{dx}\left[\frac{2x^3-5x}{\sqrt{x}}\right]$