Problem 1.

a. What is the technical definition of the *derivative of a function*?

b. What is the 'intuitive idea' or interpretation of the derivative of a function? How can we geometrically interpret the derivative of a function.

c. Compute the derivative of the function $f(x) = 3 - 5x^2$ using the definition.

d. Compute the derivative of the function $f(x) = \sqrt{x+1}$ using the definition.

e. Compute the derivative of the function $f(x) = \dfrac{x}{x+4}$ using the definition.

Problem 2.

Compute the derivative of the following function.

a. $f(x) = \dfrac{4x^3 - 7\sqrt{x}}{x^2}$

b. $g(t) = e^{-t}\sin(t)$

c. $f(t) = \cos(5t^2)$

d. $h(x) = \tan^{-1}(x)$

e. $f(x) = 3(x^2 - \ln(x))$

f. $h(x) = \dfrac{1}{\sqrt{1+x^2}}$

g. $p(x) = \cos(\cos(x))$

h. $q(x) = \dfrac{1}{1-\sin^2(x)}$

i. $t(x) = e^{-\tan(x)}$

Problem 3.

Consider the function $f(x) = \dfrac{4x-3}{7+x}$.

a. Find all vertical and horizontal asymptotes of f.

b. Find all roots of f.

c. Find the critical points, and inflection points of f.

d. For what x values is f increasing/decreasing?

e. For what x values is f concave up/down?

f. Where are the local minimum and maximum values of f?

g. Make a sketch of the graph of the function f.

Repeat for the following functions:

- $f(x) = \dfrac{x^2-1}{4x^2+9}$
- $f(x) = \dfrac{x}{x^2-9}$
- $f(x) = \dfrac{x^2}{x-2}$
- $f(x) = x\ln(x)$

Problem 4.

In this problem we study a function $f(x)$. We do not have a formula for $f(x)$, but we do know that the derivative of the function is $f'(x) = \boxed{}$.

a. Find the critical points, and inflection points of f.

b. For what x values is f increasing/decreasing?

c. For what x values is f concave up/down?

d. Where are the minimum and maximum values of f?

e. Make a sketch that shows a possible graph of the function f.

Problem 5.

a. In this problem we think about a box that has a square base, four sides, but no top. The volume of the box is 4. What should the height and width of the box be in order to minimize the amount of material used to make the base and sides?

b. Suppose instead we have a cylindrical container with no lid. The volume is 10. What should the height and radius be in order to minimize the amount of material to make the base and side?

Problem 6.

a. A 5 meter long ladder is leaning against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 2 meters per second. How fast is the top of the ladder moving right at the moment when the top of the ladder is 3 meters from the floor?

b. Suppose we have a cylinder where the radius is increasing at a rate of 3 meters per second. The volume of the cylinder remains 12 cubic meters at all times. At what rate is the height of the cylinder changing at the time when the height is 3 meters?

Problem 7.

a. Find the best linear approximation to the function $f(x) = \sqrt{x}$ centered at $x_* = 9$.

b. Find the best linear approximation to the function $f(x) = \dfrac{1}{\sqrt{x}}$ centered at $x_* = 4$.

Problem 8.

a. What is the *cumulative effect* of a function from a to b? Give both the technical definition and a geometric interpretation.

b. Explain how can we use the fundamental theorem to compute cumulative effect.

c. Compute $\displaystyle\int_1^{16} \frac{1}{\sqrt{x}}\,dx$. Simplify your answer as much as you can.

d. Compute $\displaystyle\int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}\,dx$. Simplify your answer as much as you can.