$\boxed{33.1}$

we choose $(\theta, \omega) = (0,0)$

Linearized system for $u = \theta - 0$
$v = \omega - 0$

is
$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

eigenvalues are $\lambda = \pm i$,

These agree with SHO on page 112

with $\frac{k}{m} = 1$.

$\boxed{3.3.2}$

we choose $(\theta, \omega) = (\pi, 0)$.

Linearized system with $u = \theta - \pi$

$v = \omega - 0$

is $\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$

eigenvalues are $\lambda = \pm 1$.

Thus this is a saddle!

Thus unstable.

$$\boxed{33.333333\ldots}$$

3

(1) $\sin(\theta) = \theta \underbrace{- \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \ldots\ldots}_{\text{very small if } \theta \text{ small}}$

Thus

$\sin(\theta) \approx \theta$

(2) Thus (Pendulum) becomes (approximately)

$$\frac{d\theta}{dt} = \omega \qquad \frac{d\omega}{dt} = -\frac{g}{L}\theta$$

which is same form as SHO.

we can write this as

$$\frac{d}{dt}\begin{pmatrix}\theta \\ \omega\end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{g}{L} & 0 \end{pmatrix}\begin{pmatrix}\theta \\ \omega\end{pmatrix}$$

eigenvalues:

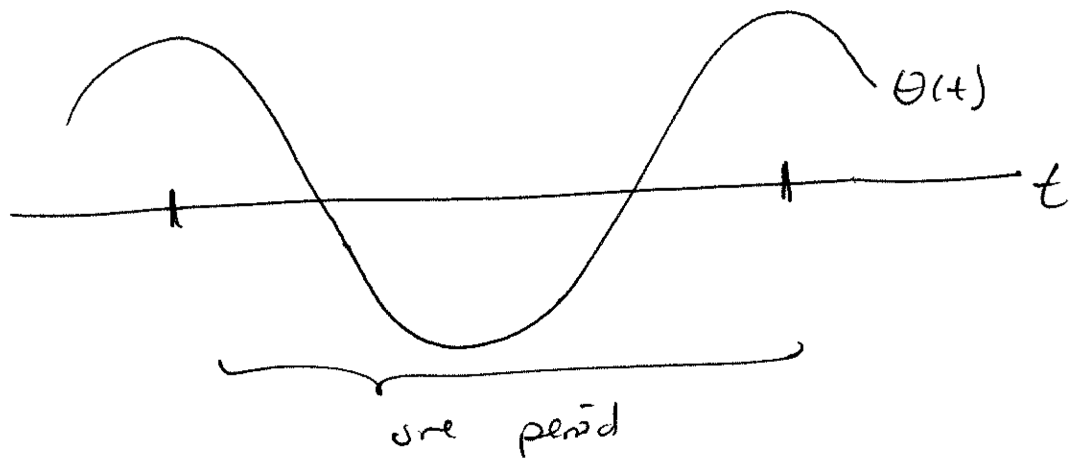
$\lambda^2 + \frac{g}{L} = 0.$

(3) so solutions are

$$\theta(t) = \alpha \cos\left(\sqrt{\frac{g}{L}}\, t\right) + \beta \sin\left(\sqrt{\frac{g}{L}}\, t\right).$$

frequency is $\sqrt{\frac{g}{L}}$.

(u) Typical solution is



we know that $\sin(\ )$ has period of 2π

so $\sin\left(\sqrt{\frac{g}{L}}\,t\right)$ has period of $2\pi\sqrt{\frac{L}{g}}$.

If we want period = 1 second then

$$2\pi\sqrt{\frac{L}{g}} = 1\,\text{s}$$

$$\frac{L}{g} = \left(\frac{1}{2\pi}\right)^2 \quad \text{seconds}^2$$

$$L = \frac{g \ \text{seconds}^2}{4\pi^2}$$

Since $g \approx 10\ \text{m/s}^2$

$$L \approx \frac{10}{4\pi^2} \text{ meters} \approx 25 \text{ centimeters}$$