$\boxed{32.1}$

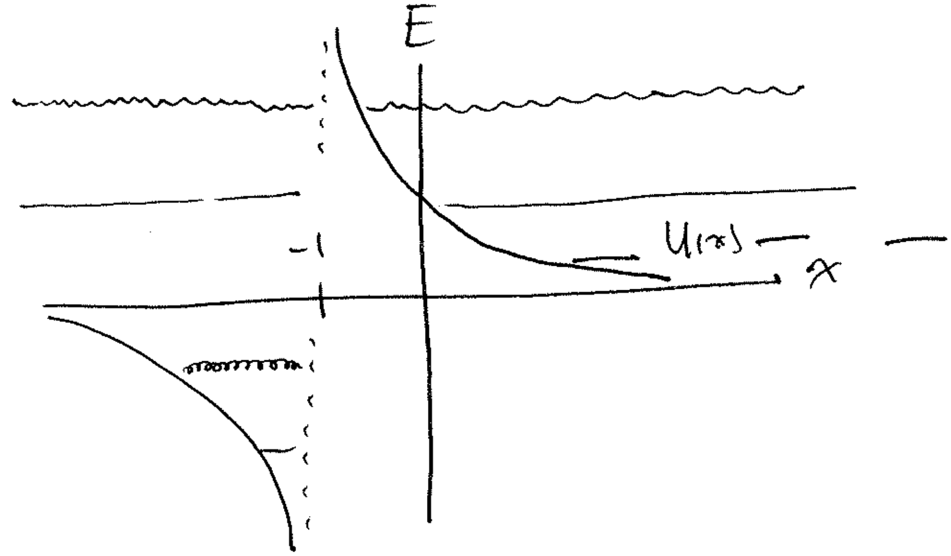


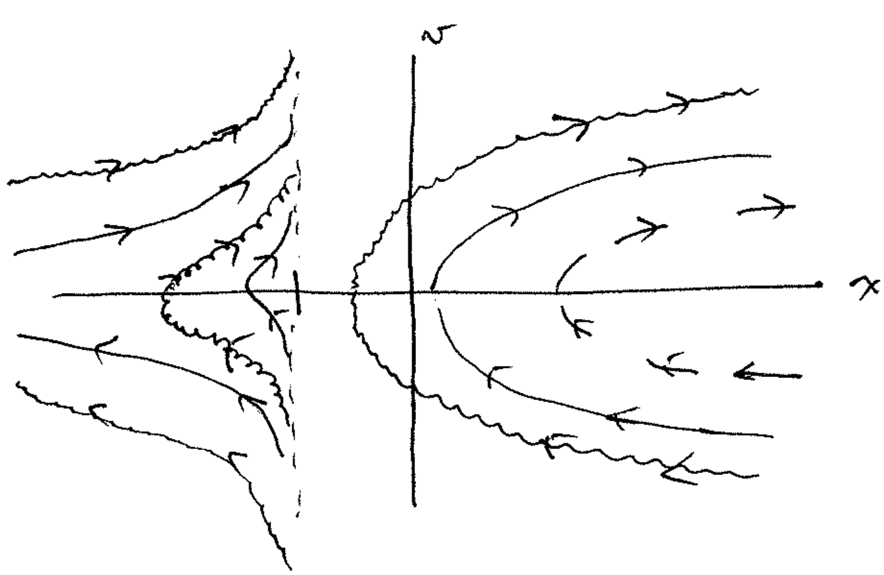
$$\frac{d^2x}{dt^2} = \frac{1}{(x+1)^2}$$

$$-U'(x) = \frac{1}{(x+1)^2} = (x+1)^{-2}$$

$$U'(x) = -(x+1)^{-2}$$

$$U(x) = (x+1)^{-1} = \frac{1}{x+1}$$



- there are no critical points of $U$ so there are no equilibrium solutions

- Solutions with $E > 0$ are unbounded:

  If $E > 0$ and $x(0) > 0$ then solutions (after possibly reflecting off potential) have $x(t) \to \infty$

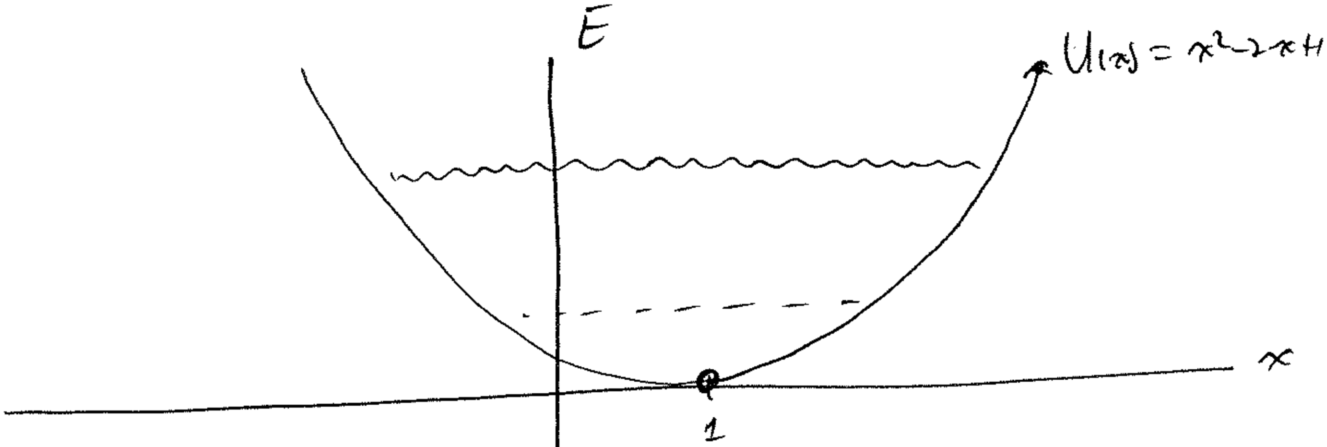
  If $E > 0$ ~~[illegible]~~ and $v(0) < 0$ then $x(t) \to -\infty$

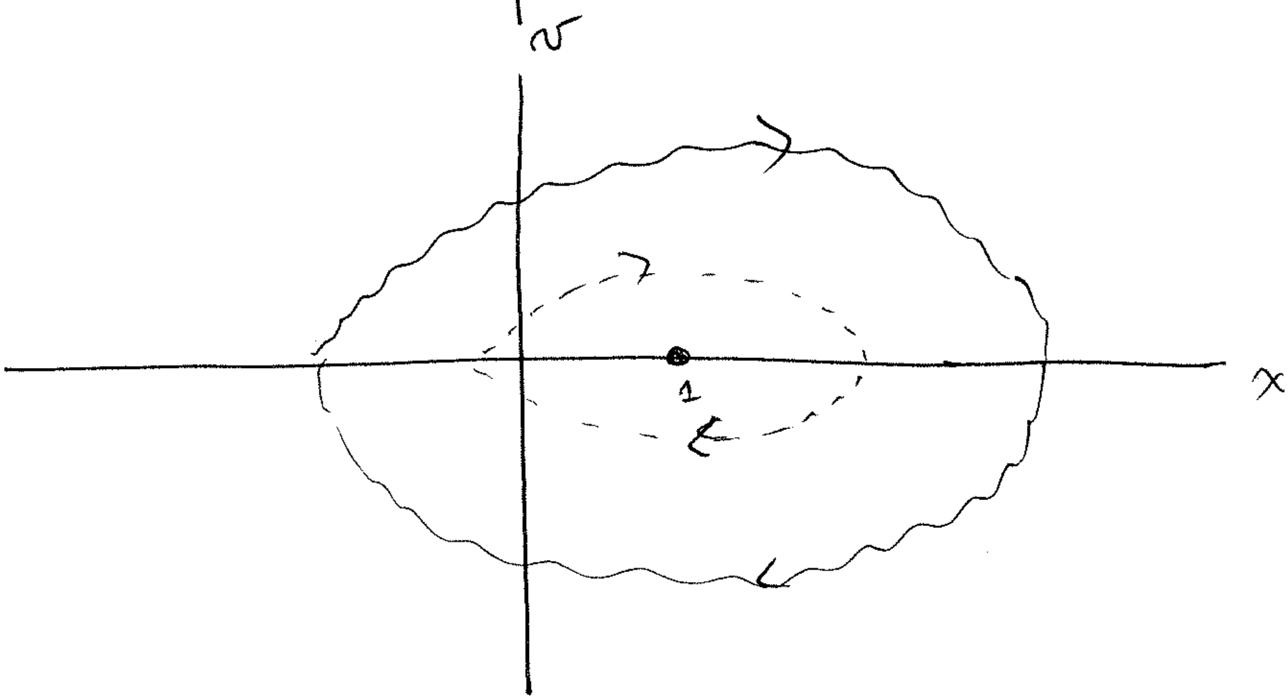
  If $E > 0$ and $v(0) > 0$ then
  $x(t) \to -1$ and
  $v(t) \to \infty$ as $t \to \infty$

  If $E < 0$ then solutions behave as $x \to -1$, $v \to +\infty$ as $t \to \infty$. This can be interpreted as a kind of blowup.
  $(v \to \infty)$

$\boxed{32.3}$





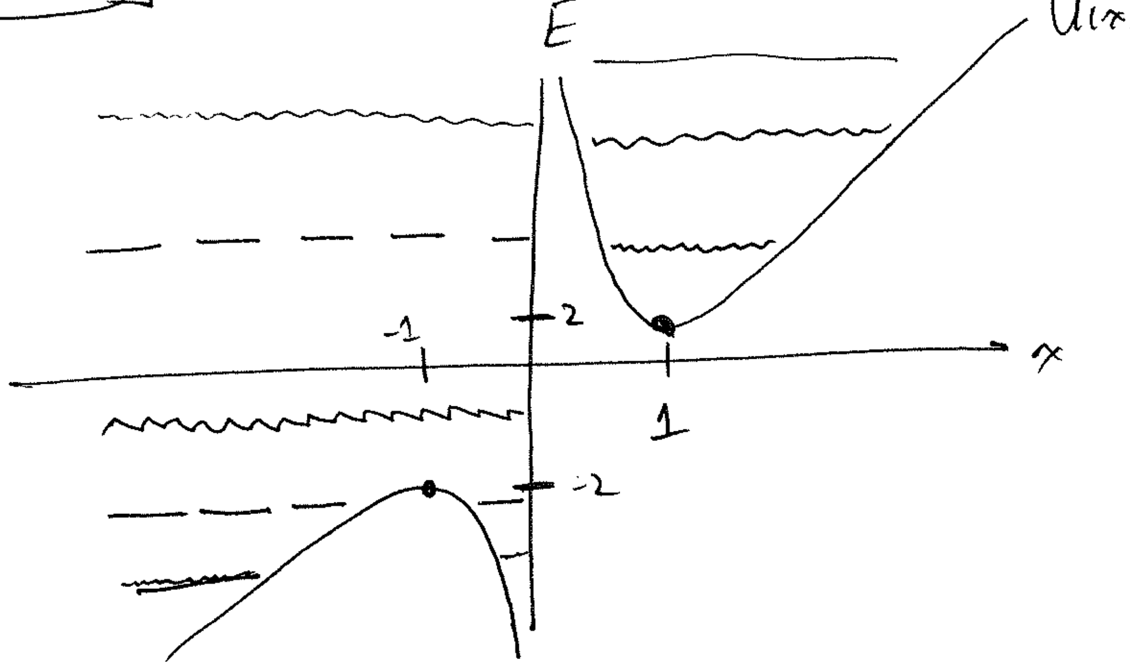



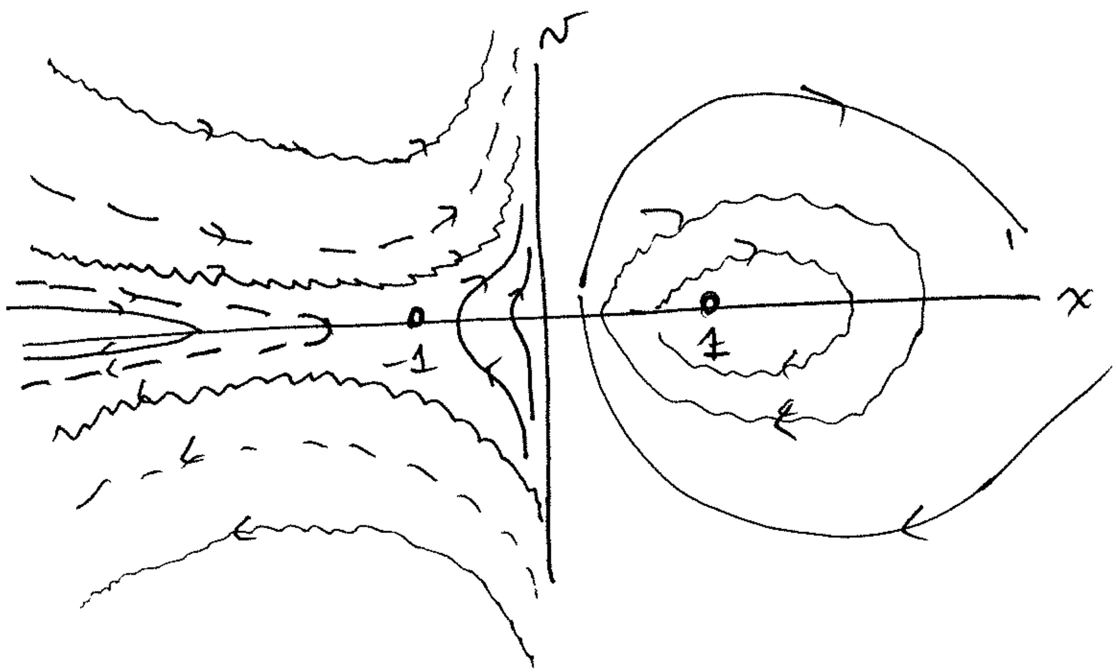

Equilibrium is at $x = 1$

All solutions oscillate about $x = 1$.

$\boxed{32.5}$

4




Critical points at ~~[illegible]~~

$$0 = U'(x) = \frac{-1}{x^2} + 1 = \frac{-1+x^2}{x^2} \longrightarrow x = \pm 1$$

$$U(1) = 2$$
$$U(-1) = -2$$

- we have a center at $x = 1$ and a saddle at $x = -1$

- If $x(0) > 0$ then solution oscillates about $x = 1$

- If $x(0) < 0$ and $E > -2$

  then:
  - $v(0) > 0$ means $x \to 0$, $v \to +\infty$
  - $v(0) < 0$ means $x \to -\infty$

- If $x(0) < 0$ and $E < -2$ then:
  - $x(0) < -1$ means $x \to -\infty$
  - $-1 < x(0) < 0$ means $x \to 0$, $v \to +\infty$

- again we see the blowup at $x \to 0$ for solutions with $x(0) < 0$, $v(0) > 0$ and either $E > -2$ or $x(0) > -1$.

$\boxed{32.7}$



If we replace $U(x) \to U(x) + c$

then
- energy diagram is shifted up by $c$
- phase diagram is not changed
- equation for $\frac{dv}{dx}$ not changed

  (because $\frac{d}{dx}[c] = 0$)

Thus it is only potential difference that matters!