

31.5

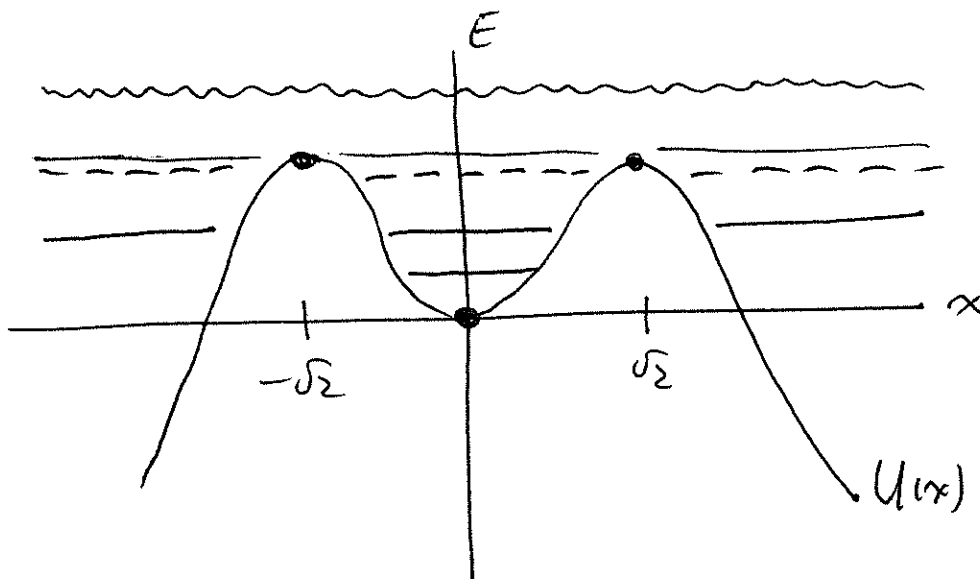
(1) $\frac{dx}{dt} = v$ $\frac{dv}{dt} = x^3 - x$

(2) $-U'(x) = x^3 - x$

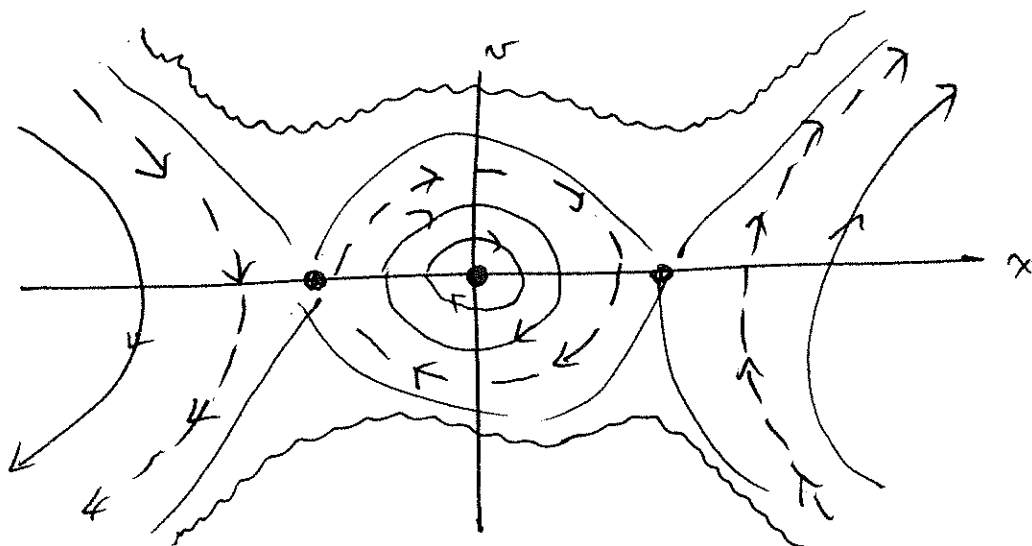
$U'(x) = x - x^3$

$U(x) = \frac{1}{2}x^2 - \frac{1}{4}x^4 = \frac{1}{2}x^2 \left(1 - \frac{1}{2}x^2\right)$
 $= \frac{1}{4}x^2(2 - x^2)$

(3)



(4)



(5) we see that we have
equilibria at $x=0, 1, -1$

$$\text{at } x=0 \quad E=0$$

$$\text{at } x=\pm 1 \quad E=\frac{1}{4}$$

Thus if $-\sqrt{2} < x(0) < \sqrt{2}$

and $E < \frac{1}{4}$ then solution
oscillates about $x=0$

if $x(0) > \sqrt{2}$ or $x(0) < -\sqrt{2}$

$$\text{or } E > \frac{1}{4}$$

then: $x(t) \rightarrow +\infty$ if $x(0) > \sqrt{2}$ and
 $E < \frac{1}{4}$

$x(t) \rightarrow +\infty$ if $x(0) > 0$ and $E > \frac{1}{4}$

$x(t) \rightarrow -\infty$ if $x(0) < -\sqrt{2}$ and $E < \frac{1}{4}$

$x(t) \rightarrow -\infty$ if $x(0) < 0$ and $E > \frac{1}{4}$

Note: there may be better ways to
organize these statements---