$\boxed{30.1}$

1

(1) A conserved quantity Q is such that $\frac{d}{dt} Q = 0$ along solutions of the equation

(2) $$\frac{d}{dt}\left[\frac{x}{y}\right] = \frac{dx}{dt}\frac{1}{y} - \frac{x}{y^2}\frac{dy}{dt}$$

$$= \frac{1}{y}\left(x^2 - xy\right) - \frac{x}{y^2}\left(xy - y^2\right)$$

$$= \frac{x^2}{y} - x - \frac{x^2}{y} + x = 0 \checkmark$$

(3) solutions must follow trajectories

$Q =$ constant

$\frac{x}{y} =$ constant

$x = (\text{constant})\, y$

these are straight lines through $(0,0)$

we have

$$\frac{dx}{dt} = x^2 - xy = x(x-y)$$

thus solutions move to right if

$x > 0$ and $x > y$

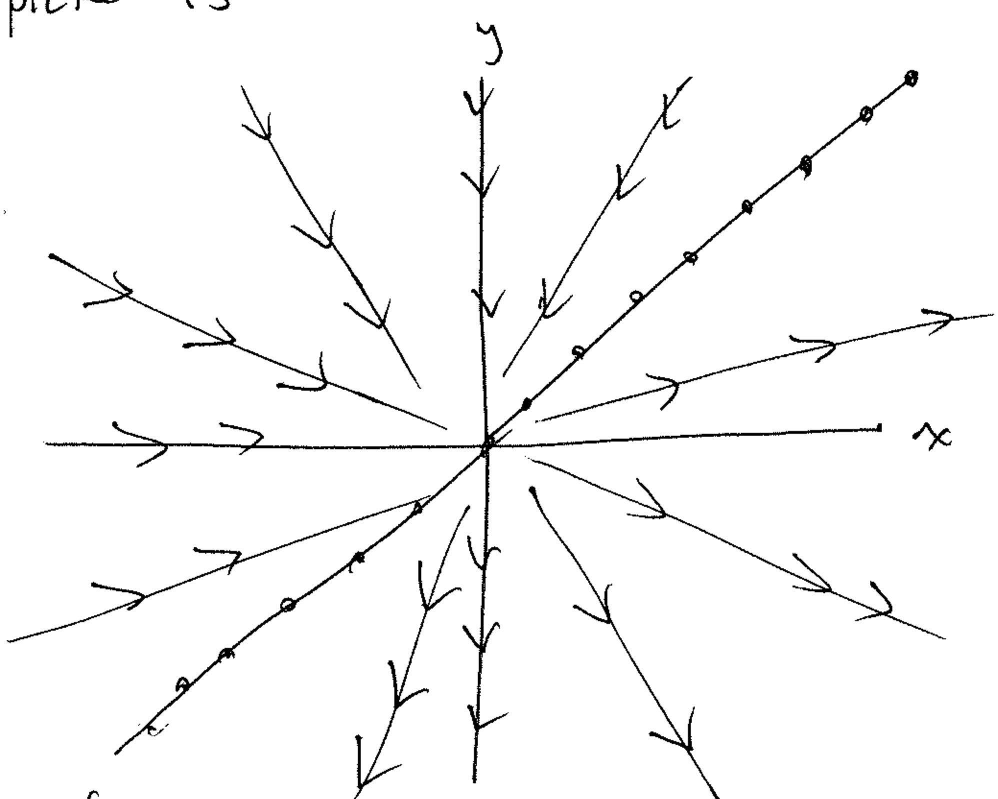
or $x < 0$ and $x < y$

and solutions move to left if

$x > 0$ and $x < y$

or $x < 0$ and $x > y$

so picture is



line of
equilibria along $x=y$

$\boxed{30.2}$

(1) we know solutions travel along curves $\frac{1}{2}v^2 + \frac{1}{2}x^2 = \text{constant}$

(2) $E = \frac{1}{2}(6)^2 + \frac{1}{2}(4)^2 = \frac{36}{2} + \frac{16}{2} = 18 + 8 = 26$

Thus solutions always satisfy

$$\frac{1}{2}x^2 + \frac{1}{2}v^2 = 26$$

$$x^2 + v^2 = 52$$

and solutions traverse a circle (centered @ (0,0)) of radius $\sqrt{52}$ in phase space.

The largest x gets is $\sqrt{52}$.

$\boxed{30.3}$

4

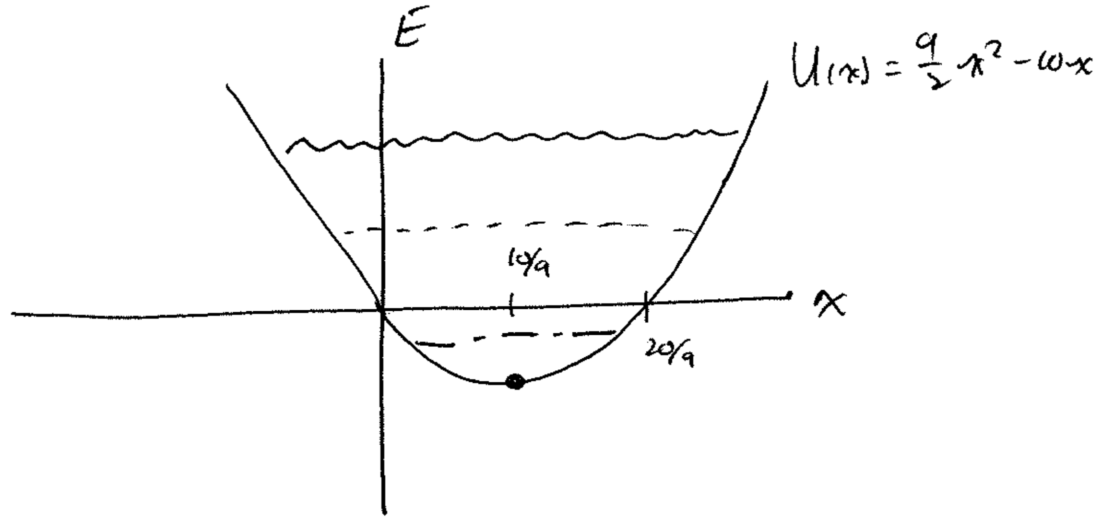
(1) $\frac{dx}{dt} = v \qquad \frac{dv}{dt} = 10 - 9x$

(2) $\frac{d}{dt}\left[\frac{1}{2}v^2 + \frac{9}{2}x^2 - 10x\right]$

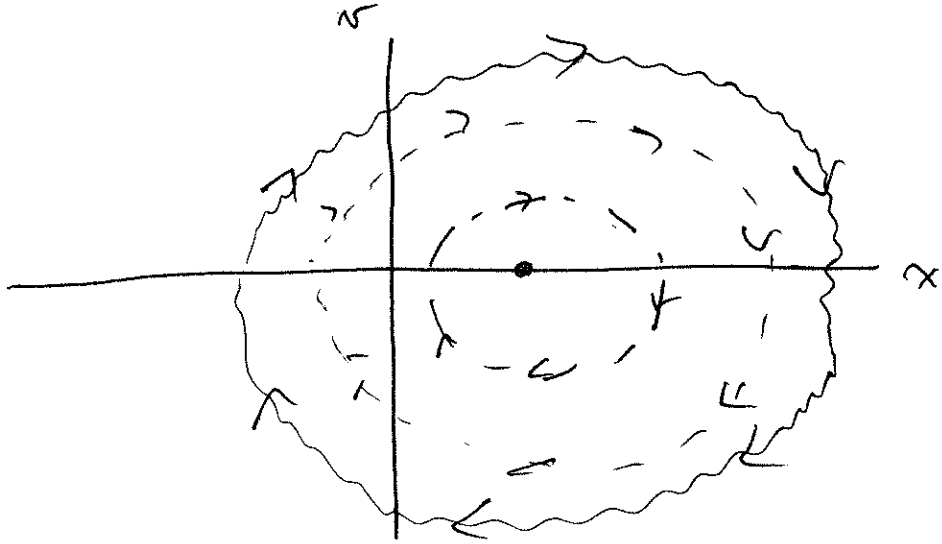
$= v\frac{dv}{dt} + 9x\frac{dx}{dt} - 10\frac{dx}{dt}$

$= v(10-9x) + 9xv - 10v = 0$ ✓

(3)



(4)



(5) Solutions oscillate, with oscillations centered about

$x = \frac{10}{9}$.

(6) homogeneous solution is

$$y_h(t) = \alpha \cos(3t) + \beta \sin(3t)$$

look for $y_p(t) = A$. plug in

or $9A = 10 \rightsquigarrow A = \frac{10}{9}$

Thus $y_p(t) = \frac{10}{9}$ and general solution is

$$y(t) = \alpha \cos(3t) + \beta \sin(3t) + \frac{10}{9}$$

This precisely matches the prediction that solutions oscillate about $x = \frac{10}{9}$!