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29.1

(1) look for $y = e^{\lambda t}$.

plug in to get $\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

general solution

$y = \alpha e^{3t} + \beta e^{t}$

(2) from (1) homogeneous solution is

$y_h(t) = \alpha e^{3t} + \beta e^{t}$

we look for y_p of form

$A + Bt + Ce^{2t}$

plugging in yields

$4Ce^{2t} - 4(B + 2Ce^{2t}) + 3(A + Bt + Ce^{2t})$

$= 1 + t + e^{2t}$

$(-4B + 3A) + (3B)t + (4C - 8C + 3C)e^{2t}$

$= 1 + t + e^{2t}$

Thus

$$-4B + 3A = 1$$
$$3B = 1 \longrightarrow B = \frac{1}{3}$$
$$-C = 1 \longrightarrow C = -1$$

$$\longrightarrow -4\left(\frac{1}{3}\right) + 3A = 1$$
$$-\frac{4}{3} + 3A = 1$$
$$3A = \frac{7}{3}$$
$$A = 7/9$$

So $y_p(t) = \frac{7}{9} + \frac{1}{3}t - e^{2t}$

and

$$y(t) = \alpha e^{3t} + \beta e^{t} + \frac{7}{9} + \frac{1}{3}t - e^{2t}.$$

(3) plugging in $e^{\lambda t}$ yields

$$\lambda^2 - 4\lambda + 4 = 0$$
$$(\lambda - 2)^2 = 0$$

So e^{2t} is only eigen soluti

By t trick we have $y(t) = \alpha e^{2t} + \beta t e^{2t}$.

(4) homogeneous equat is

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = 0$$

plug in $e^{\lambda t}$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda - 4)(\lambda - 1) = 0$$

homogeneous soluti

$$y_h(t) = \alpha e^{4t} + \beta e^t$$

Since forcing is same type as $y_h(t)$

we guess

$$y_p = Ate^{4t} + Bte^t$$

plug in:

$$\frac{dy_p}{dt} = Ae^{4t} + 4Ate^{4t} + Be^t + Bte^t$$

$$\frac{d^2y}{dt^2} = 4Ae^{4t} + 4Ae^{4t} + 16Ate^{4t}$$
$$+ Be^t + Be^t + Bte^t$$

Thus

$$\begin{aligned}
&\left[4A + 4A - 5A\right]e^{4t} \\
+ &\left[16A - 20A + 4A\right]te^{4t} \\
+ &\left[B + B - 5B\right]e^{t} \\
+ &\left[B - 5B + 4B\right]te^{t} = e^{4t}
\end{aligned}$$

This implies $-A = 1$, $-3B = 0$

So $y_p(t) = -te^{4t}$

and

$$y(t) = \alpha e^{4t} + \beta e^{t} - te^{4t}$$

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(5) homogeneous soln is

$$y_h(t) = \alpha \cos(3t) + \beta \sin(3t)$$

guess $y_p(t) = A\sin(2t)$

plug in

$$-4A \sin(2t) + 9A\sin(2t) = \sin(2t)$$

$$5A \sin(2t) = \sin(2t)$$

Thus $A = \frac{1}{5}$ and

$$y(t) = \alpha \cos(3t) + \beta \sin(3t) + \frac{1}{5} \sin(2t)$$

(6) homogeneous solution is

$$y_h(t) = \alpha \cos(3t) + \beta \sin(3t)$$

guess

$$y_p(t) = At\cos(3t) + Bt\sin(3t)$$

compute

$$\frac{dy_p}{dt} = A\cos(3t) - 3At\sin(3t)$$
$$B\sin(3t) + 3Bt\cos(3t)$$

$$\frac{d^2y_p}{dt^2} = -3A\sin(3t) - 3A\sin(3t) - 9At\cos(3t)$$
$$+3B\cos(3t) + 3B\cos(3t) - 9Bt\sin(3t)$$

equation becomes

$$\left[6B \right]\cos(3t) + \left[-9A + 9A\right]t\cos(3t)$$

$$\left[-6A \right]\sin(3t) + \left[-9B + 9B\right]t\sin(3t)$$

$$= 3\cos(3t)$$

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Thus we need

$$6B = 3 \qquad \text{and} \qquad -6A = 0$$

So

$$\boxed{y(t) = \alpha \cos(3t) + \beta \sin(3t) + \frac{1}{2} t \sin(3t)}$$