Day 1 solutions

Solution to Exercise 1.1.

(1) $\frac{dy}{dt} = 0.05y$
(2) $\frac{dy}{dt} = 0.05y$ (with the initial condition of $y(0) = 1$)
(3) Assuming $y(t)$ is the value of the investment in thousands of dollars, we have:
$$\frac{dy}{dt} = 0.07y + 1,$$
with the initial condition of $y(0) = 1$.
(4) Assuming $y(t)$ is the value of the loan in thousands of dollars, we have:
$$\frac{dy}{dt} = 0.06y - 1.$$
(5) Assuming $y(t)$ is the value of the loan in thousands of dollars, we have:
$$\frac{dy}{dt} = 0.06y - 1,$$
with the initial condition $y(0) = 20$.
(6) Assuming $y(t)$ is the fish population size, we have:
$$\frac{dy}{dt} = ky\left(1 - \frac{y}{N}\right) - 12H.$$
(7) Assuming $y(t)$ is the fish population size, we have:
$$\frac{dy}{dt} = ky\left(1 - \frac{y}{N}\right) - 0.25y.$$
(8) Assuming $y(t)$ is the population size, we have
$$\frac{\frac{dy}{dt}}{y} = ce^{-rt}, \quad \text{i.e.} \frac{dy}{dt} = cye^{-rt},$$
for some positive constants c and r. Other solutions are possible.
(9) This is a system of differential equations
$$\frac{\frac{dx_A}{dt}}{x_A} = \frac{C_A}{x_B}, \quad \frac{\frac{dx_B}{dt}}{x_B} = \frac{C_B}{x_A};$$

the parameters C_A and C_B are some constant parameters. Overall the model is:

$$\frac{dx_A}{dt} = C_A \frac{x_A}{x_B}, \quad \frac{dx_B}{dt} = C_B \frac{x_B}{x_A}.$$