Recap: linear systems in 2D and 3D

Geometric interpretation of equations in 2 and 3 variables

- equations of the form $ax + by = f$ correspond to a line in 2D space
- equations of the form $ax + by + cz = f$ correspond to a plane in 3D space
- the right hand side (that is, the number f) is zero precisely when the line/plane passes through the origin

Systems of equations

- Systematically find all solutions using Gauss' method: add equations together, multiply equations by a constant, swap the order of equations
- Strategy tip: First isolate x, then y, etc.
- Geometrically interpret space of solutions: none, point, line, plane

Linear subspaces in 3D space

- Options are the point $\mathbf{0}$, a line (through $\mathbf{0}$), a plane (through $\mathbf{0}$), all of 3D space
- A linear subspace can be described as the *span* of certain vectors
- If a collection of vectors spaces a subspace, and if the collection is not redundant, then the collection is called a *basis* for the subspace

Redundant vectors in 3D space

- Three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in 3D are redundant if they lie on the same plane
- This is equivalent to there being α, β, γ—not all zero—such that

  $$\alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \gamma\mathbf{v}_3 = 0.$$

  This is a system of equations we can try to solve
- Geometric interpretation: Round trip