Math 225 **Exam 1 – Practice** Fall 2015

Here are some problems to help you practice for the first exam.

Problem 1. Give careful definitions of the following terms. (Be sure to give the "formal" definition and not a preliminary "intuitive" definition.)

A. A *subspace* of $\mathbb{R}^n$.

B. A *linear transformation*.

C. The *kernel and range* of a linear transformation.

D. A *linearly independent* collection of vectors.

E. The *span* of a collection of vectors.

F. A *basis* for a subspace.

G. A *consistent* system of equations.

Problem 2.

A. State the Rank-Nullity Theorem.

B. Suppose $f\colon \mathbb{R}^3 \to \mathbb{R}^4$ has a one dimensional kernel. What else can you say about f?

C. Suppose $f\colon \mathbb{R}^3 \to \mathbb{R}^2$. What can you say about the kernel of f?

D. Suppose $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ has $\ker\{\mathbf{0}\}$. Discuss the solvability of $f(\mathbf{v}) = \mathbf{r}$.

E. Suppose $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ has $\operatorname{ran}(f) = \mathbb{R}^2$. Discuss the solvability of $f(\mathbf{v}) = \mathbf{r}$.

F. Suppose $f\colon \mathbb{R}^n \to \mathbb{R}^m$. Under what conditions is $f(\mathbf{v}) = \mathbf{0}$ solvable?

Problem 3. Consider the system of equations

$$\begin{aligned}
12x - 9y + 3z - 2w &= 57\\
-12x + 9y + 3z + 4w &= -63\\
8x - 6y - 3z - 3w &= 43\\
-8x + 6y - 3z + w &= -37
\end{aligned}$$

(Note: The numbers here aren't the best... I'm saving the better numbers for the actual exam!)

A. Find the space of solutions to this system. Describe the solution space geometrically.

B. Express the system of equations in terms of a linear transformation f. State the domain and codomain of f.

C. What is the kernel of f? Describe using a basis, and also give a geometric description.

D. What is the range of f? Describe using a basis and also give a geometric description.

Problem 4. Consider the transformation $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ 6x + 2y \end{pmatrix}$

A. What is the determinant of this transformation? Give a geometric interpretation.

B. Find the eigenvalues of this transformation. For each eigenvalue, find the corresponding eigenspace.

C. Explain how we know that f has an inverse transformation. Find the formula for f^{-1}.