

## TOPIC 2

### Matrices and Gauss' method

#### Augmented matrices

- Write systems of equations in an *augmented matrix*: each row is an equation, dashed line represents equal sign
- Gauss' method: Equation operations  $\leftrightarrow$  row operations
- Solving for each variable  $\leadsto$  *identity matrix*
- Strategy tip  $\leftrightarrow$  reduced row echelon form (RREF):
  - Each row begins with a *leading 1*, unless it is a row of zeroes
  - If a column contains a leading 1, the remainder of the column is zeroes
  - leading 1s sometimes called “pivots”
- Converting back to variables
- *free variables* and *dependent variables*  $\leadsto$  basis for space of solutions

#### Vocabulary:

- a system of equations is *homogeneous* if the right side of all equations is zero; otherwise it is an *inhomogeneous* systems of equations
- a system of equations is *consistent* if there is at least one solution (i.e., no contradictions arise); otherwise it is an *inconsistent system*
- a system of equations is *independent* if no equation can be obtained from the others; otherwise it is a *dependent system*.

#### Fun facts:

- All homogeneous equations are consistent—one solution is all zeroes!
- Inconsistent systems lead to  $0 =$  (something not zero) in RREF
- Dependent systems lead to rows of zeroes in RREF

**Exercise 2.1.** Construct an example of an inconsistent system. Explain, using augmented matrices and Gauss' method, how we know that it is inconsistent.

**Exercise 2.2.** Construct an example of a consistent, inhomogeneous system. Use augmented matrices and Gauss' method to describe all solutions.

**Exercise 2.3.** Use augmented matrices and Gauss' method to find all solutions to the system

$$2x + 3y - z = 15$$

$$x - 4y - 6z = 2$$

$$3x + y + 5z = -1$$

Is the system dependent or independent? Homogeneous or inhomogeneous? Consistent or inconsistent?

**Exercise 2.4.** Repeat for the system

$$x + y + 2z = 7$$

$$3x + 5y + 8z = 25$$

$$3x + y + 2z = 11$$

**Exercise 2.5.** Repeat for the system

$$2x + 3y - z = 0$$

$$x - 4y - 6z = 0$$

$$3x + y - 5z = 0$$

**Exercise 2.6.** Repeat for the system

$$x + 2y - 5z = 3$$

$$2x - 2y + 14z = -4$$

$$3x + 2y + z = 1$$

**Exercise 2.7.** Repeat for the system

$$x + 2y - 5z = 24$$

$$2x - 2y + 14z = -66$$

$$3x + 2y + z = -4$$