ASSIGNMENT 19

Definite integrals

Exercise 19.1. Here we review the main concepts of this course.

(1) What is the *derivative* of a function? Give both an "intuitive description" and a technical definition (involving limits).

(2) What is the *antiderivative* of a function?

(3) What is the *definite integral* of a function over some interval? Give both an "intuitive description" and a technical definition (involving limits).

(4) How does the First Fundamental Theorem of Calculus relate derivatives and definite integrals?

(5) How does the Second Fundamental Theorem of Calculus relate derivatives and definite integrals?

(6) What is one geometric interpretation of the derivative of a function? What is one geometric interpretation of the definite integral of a function?

Exercise 19.2. Compute the following definite integrals

(1) $\displaystyle\int_0^3 (4x^2 - 3x + 10)\,dx$

(2) $\displaystyle\int_0^\pi \sin x\,dx$

(3) $\displaystyle\int_0^\pi \cos x\,dx$

(4) $\displaystyle\int_0^1 e^{-x}\,dx$

(5) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}\,dx$

(6) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$

Exercise 19.3. In this problem you explore the idea of *average value*.

(1) Suppose Paul goes for a walk, starting at time $t = 0$, ending at time $t = 3$, and with velocity given by the function $v(t) = 10 - 6t^2$. At the end of his walk, how far is Paul from his original starting point?

(2) Suppose Paul wanted to walk at a constant velocity, but wants to end up at the same point at the end of the time interval. How fast should he walk?

(3) I claim that the number you computed in the previous part can be thought of as the *average value* of the velocity function $v(t)$ over the time interval from $t = 0$ to $t = 3$. Explain why this is reasonable.

(4) In general, how should we define the average value of a function $f(x)$ over the interval from $x = a$ to $x = b$?