ASSIGNMENT 18

Antiderivatives

- If $f(x)$ is some function, then an antiderivative $F(x)$ is a function such that $F'(x) = f(x)$.
- Since the derivative of a constant is zero, the *general antiderivative* includes a constant.
- Example: If $f(x) = x^2$, then the general anti-derivative is $F(x) = \frac{1}{3}x^3 + C$, where C is some constant.
- Basic principles: Remembering rules backwards, reverse engineering.

$$
\begin{aligned}
f(x) &= \cos x &&\rightsquigarrow && F(x) = \sin x + C \\
f(x) &= \sin x &&\rightsquigarrow && F(x) = -\cos x + C \\
f(x) &= x^7 &&\rightsquigarrow && F(x) = \frac{1}{8}x^8 + C
\end{aligned}
$$

- Notation:

$$F(x) = \int f(x)\,dx$$

We'll soon see where this notation comes from. The dx tells us what the variable is.

- Examples

$$\int x^2\,dx = \frac{1}{3}x^3 + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int 7\,dx = 7x + C$$

$$\int (6x^2 - \sqrt{x})\,dx = 2x^3 - \frac{2}{3}x^{3/2} + C \qquad \int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$$

- Basic rules

$$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int c\,f(x)\,dx = c\int f(x)\,dx$$

Exercise 18.1. Find the general antiderivatives of the following functions:

(1) $f(x) = 12x^3 - 9x + 17$

(2) $f(x) = \frac{5}{\sqrt{x}}$

(3) $f(x) = 4\sin(x)$

(4) $f(x) = 4\sin(4x)$

(5) $f(x) = e^{-x}$

(6) $f(x) = \frac{1}{x}$

Exercise 18.2. Find the following

(1) $\displaystyle\int \frac{1}{1+x^2}\,dx$

(2) $\displaystyle\int \frac{1-x^2}{x}\,dx$ (Hint: Break in to two fractions.)

(3) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$

(4) $\displaystyle\int \frac{1}{\cos^2 x}\,dx$

(5) $\displaystyle\int \frac{3}{x^5}\,dx$

Exercise 18.3. Here we use an algebraic trick to find an antiderivative.

(1) Use common denominators to show that $\dfrac{1}{1+x} + \dfrac{1}{1-x} = \dfrac{2}{1-x^2}$

(2) Find $\displaystyle\int \frac{2}{1-x^2}\,dx$