ASSIGNMENT 15

Optimization and related-rates problems

Exercise 15.1. Compute the derivative of the following functions

(1) $f(x) = \sqrt{1 + e^{2x}}$

(2) $f(x) = \ln(1 + x^2)$

(3) $f(x) = e^{-x} \cos x$

(4) $f(x) = \sin^{-1}(\ln x)$

(5) $f(x) = \frac{1}{2}x^2 e^{-x^2}$

(6) $f(x) = \dfrac{3x}{4 + x^2}$

Exercise 15.2. (Famous fence problem) Suppose that a rectangular region is created using 100 meters of rope as the boundary.

(1) Let W be the width of the region and A be the area of the region. Find a formula that relates W and A.

(2) Suppose now that the width W is changing in time; find a relationship between $\frac{dW}{dt}$ and $\frac{dA}{dt}$. If $\frac{dW}{dt} = 7$ meters per second, at what rate is the area changing when the width is 4?

(3) Let us now change perspective and view A as a function of W. Make a plot of this function on the W-A axes. What value of W makes A the largest?

Exercise 15.3. (Famous soup can problem) A company is experimenting with designs for a cylindrical can with a volume of 100 cubic centimeters.

(1) Let H be the height of the can, let R be the radius of the can, and let A be the area of the side wall of the can. Find a relationship between H and A, and a relationship between H and R.

(2) Suppose that the radius R is changing in time. Find a relationship between the rate of change in R and the rate of change in H. If at some moment the radius is 2 centimeters and the radius is increasing at 3 centimeters per minute, at what rate is the height changing?

(3) Now we view the area A as a function of H. In order to optimize the amount of material used to construct the can, we want to minimize A. Find the dimensions (height and radius) of the can that has the smallest A value.