ASSIGNMENT 7

First and second derivatives

Reading: Sections 3.3–3.6 in Smith–Minton have lots of details.

Physical interpretations of derivatives

- our function $f(t)$ represents location
- the first derivative $f'(t)$ represents velocity; an alternate notation is $\frac{df}{dt}$
- the second derivative $f''(t)$ represents acceleration; alternate notation $\frac{d^2f}{dt^2}$

Interpreting the associated pictures

- Consider the graph of $f(t)$, where height is the value of f at input t
- $f'(t)$ is the rate of change of f; on graph of f this corresponds to slope
- $f''(t)$ tells us the change in $f'(t)$; on the graph of f this corresponds to concavity

Important inputs

- *roots* of f occur at inputs where $f(t) = 0$. Pictorially, these are inputs where the graph of f crosses the input axis
- *critical points* of f occur at inputs where $f'(t) = 0$. Pictorially, these are inputs where the graph of f has slope zero. All maxima/minima occur at critical points.
- *inflection points* of f occur at inputs where $f''(t) = 0$. Pictorially, these are inputs where the concavity of f is zero. All changes in concavity occur at inflection points.

Second derivative test

- Suppose t_* is a critical input.
- If $f''(t_*) > 0$ then f has a *local minimum* at t_*
- If $f''(t_*) < 0$ then f has a *local maximum* at t_*
- If $f''(t_*) = 0$ then we need to look more closely

Examples

- $f(t) = 2t^3 - 3t^2 - 12t + 18$
- $f(x) = x^3 - 27x$

Exercise 7.1. For each function below do the following:

- Find the roots, critical points, and inflection points
- Determine where the function is increasing/decreasing
- Determine where the function is concave up/down
- Find local minima and maxima
- Make a rough sketch of the graph of the function; indicate the critical and inflection points on the sketch.

(1) $f(t) = t^3 - 6t^2 - 9t - 54$

(2) $f(t) = t^3 - 6t^2 - 15t + 90$

(3) $f(t) = -2t^3 + 6t^2 + 48t - 144$

(4) $f(t) = -2t^3 + 18t^2 - 54t + 486$