

**Exercise 7.1.** For each function below do the following:

- Find the roots, critical points, and inflection points
- Determine where the function is increasing/decreasing
- Determine where the function is concave up/down
- Find local minima and maxima
- Make a rough sketch of the graph of the function; indicate the critical and inflection points on the sketch.

(1)  $f(t) = t^3 - 6t^2 + 9t - 54$

(2)  $f(t) = t^3 - 6t^2 - 15t + 90$

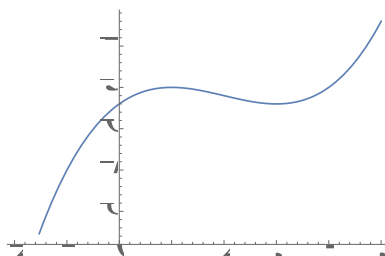
(3)  $f(t) = -2t^3 + 6t^2 + 48t - 144$

(4)  $f(t) = -2t^3 + 18t^2 - 54t + 486$

**Solution.**

(1) We have the following:

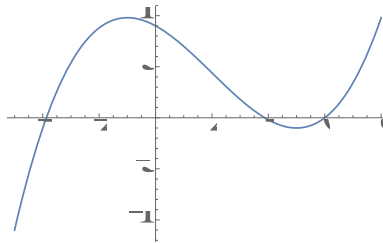
- Roots at  $t = 6$ . Critical points at  $t = 1$  and  $t = 3$ . Inflection point at  $t = 2$ .
- Increasing when  $t < 1$  or  $t > 3$ . Decreasing when  $1 < t < 3$
- Concave up when  $t > 2$  Concave down when  $t < 2$
- Local maxima at  $t = 1$  Local minima at  $t = 3$
- Plot



(2) We have the following:

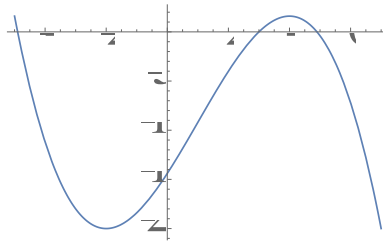
- Roots at  $t = 6$ ,  $t = \sqrt{15}$ ,  $t = -\sqrt{15}$ . Critical points at  $t = -1$  and  $t = 5$ . Inflection point at  $t = 2$ .
- Increasing when  $t < -1$  or  $t > 5$ . Decreasing when  $-1 < t < 5$
- Concave up when  $t > 2$  Concave down when  $t < 2$
- Local maxima at  $t = -1$  Local minima at  $t = 5$

- Plot



(3) We have the following:

- Roots at  $t = 3, t = -2\sqrt{6}, t = 2\sqrt{6}$ . Critical points at  $t = -2$  and  $t = 4$ .  
Inflection point at  $t = 1$ .
- Increasing when  $-2 < t < 4$ . Decreasing when  $t < -2$  or  $t > 4$
- Concave up when  $t < 1$  Concave down when  $t > 1$
- Local maxima at  $t = 4$  Local minima at  $t = -2$
- Plot



(4) We have the following:

- Roots at  $t = 9$ . Critical points at  $t = 3$ . Inflection point at  $t = 3$ .
- Increasing never. Decreasing except when  $t = 3$
- Concave up when  $t < 3$  Concave down when  $t > 3$
- No maxima or minima
- Plot

