ASSIGNMENT 5

Derivatives

Reading: §2.1, 2.2 of Smith-Minton

Summary of class notes:

- Velocity function called the *derivative* of f
- Various notations include $f'(t)$, $\dfrac{df}{dt}$, $\dfrac{d}{dt}\left[f(t)\right]$, etc.
- Equivalent definitions:

$$f'(t) = \lim_{T \to t} \left[\frac{f(T) - f(t)}{T - t} \right]$$

$$f'(t) = \lim_{h \to 0} \left[\frac{f(t+h) - f(t)}{h} \right]$$

$$f'(t) = \lim_{\Delta t \to 0} \left[\frac{f(t+\Delta t) - f(t)}{\Delta t} \right]$$

- Basic examples: $f(t) = t^2$, $f(x) = \frac{1}{x}$, $g(z) = \sqrt{z}$.
  Write as

$$\frac{d}{dt}\left[t^2\right] = 2t \qquad \frac{d}{dt}\left[\frac{1}{t}\right] = -\frac{1}{t^2} \qquad \frac{d}{dt}\left[\sqrt{t}\right] = -\frac{1}{\sqrt{t}}$$

- More complicated examples:

$$f(t) = 3t^2 + 2t - 5, \qquad f(x) = \frac{3}{2-x}, \qquad g(z) = \sqrt{5 - z}$$

- Plot both $f(t)$ and $f'(t)$.

Exercise 5.1. For each function below you need to (i) compute the derivative and (ii) sketch a plot of both the function and its derivative.

(1) $f(x) = 2x^2 - 31$

(2) $f(t) = \dfrac{3}{t+5}$

(3) $f(t) = \dfrac{3}{\sqrt{t+5}}$

(4) $f(x) = \dfrac{x}{x+1}$

Exercise 5.2. Here we analyze the absolute value function $f(x) = |x|$.

(1) Show directly (using computations involving limits) that $f'(x) = 1$ for any $x > 0$.
(2) Show directly (using computations involving limits) that $f'(x) = -1$ for any $x < 0$.
(3) Show (using computations involving limits) that the limit definition of $f'(0)$ is not defined.
(4) Sketch the graph of f and explain why it makes sense that f does not have a well-defined derivative at $x = 0$.

If you want more practice, I suggest §2.2 problems 5–12 in Smith–Minton.