ASSIGNMENT 4

Limits and velocity

Reading: §2.1 of Smith-Minton

Exercise 4.1. Write a couple of sentences describing the relationship between a function $f(t)$ measuring distance and the corresponding velocity function $v(t)$. You should use phrases such as "rate of change" as well as "geometric interpretation."

Exercise 4.2. Here you study in depth the function $f(t) = 3t^2$.

(1) Compute the velocity of f at $t = -2$, $t = -1$, $t = 0$, $t = 1$, $t = 2$.

(2) Make a plot of the function $f(t)$ on the interval $-2 \le t \le 2$.

(3) Make a rough sketch of the velocity function $v(t)$, using the points you computed in part (1).

Exercise 4.3. *The purpose of this exercise is to practice being careful with notation.* Consider the function $f(x) = \dfrac{2x^2 - x - 3}{x^2 - 5x + 6}$. Complete the following, being very careful with notation.

(1) First we factor the numerator and denominator, writing the function as

$$f(x) = \frac{(2x-3)(\qquad)}{(x-2)(\qquad)}.$$

We have roots at $x = 3/2$ and $x =$ ____, and we have vertical asymptotes at $x = 2$ and $x =$ ____.

(2) We analyze the vertical asymptote at $x = 2$. First, we examine what happens as x approaches from the right:

$$\lim_{x \to 2^+} [f(x)] = \lim_{x \to 2^+} \left[\frac{(2x-3)(\qquad)}{(x-2)(\qquad)} \right] \qquad \text{looks like} \quad \frac{(1)(3)}{(0^+)(-1)} < 0$$
$$= -\infty.$$

Next, we examine what happens as x approaches from the left:

$$\lim_{x\to 2^-} [f(x)] = \lim_{x\to 2^-}\left[\frac{(2x-3)(\quad\quad)}{(x-2)(\quad\quad)}\right] \qquad \text{looks like} \quad \frac{(1)(\quad)}{(0^-)(\quad)} > 0$$
$$= +\infty.$$

(3) We analyze the vertical asymptote at $x =$ ____. First, we examine what happens as x approaches from the right:

$$\lim_{x\to \_\_} [f(x)] = \lim_{x\to \_\_}\left[\frac{(2x-3)(\quad\quad)}{(x-2)(\quad\quad)}\right] \qquad \text{looks like} \quad \frac{(\quad)(\quad)}{(\quad)(0^+)} \quad 0$$
$$= \_\_\_\_.$$

Next, we examine what happens as x approaches from the left:

$$\lim_{x\to \_\_} [f(x)] = \lim_{x\to \_\_}\left[\frac{(2x-3)(\quad\quad)}{(x-2)(\quad\quad)}\right] \qquad \text{looks like} \quad \frac{(\quad)(\quad)}{(\quad)(\quad)} \quad 0$$
$$= \_\_\_\_.$$

(4) We determine if there is a horizontal asymptote by computing

$$\lim_{x\to\infty} [f(x)] = \lim_{x\to\infty}\left[\frac{2x^2-x-3}{x^2-5x+6}\right]$$
$$= \lim_{x\to\infty}\left[\frac{2x^2-x-3}{x^2-5x+6}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right]$$
$$= \lim_{x\to\infty}\left[\frac{\qquad}{\qquad}\right] \qquad \text{looks like} \quad \frac{\qquad}{1-0+0}$$
$$= 2$$

The same computation holds as $x \to -\infty$. Thus we have a horizontal asymptote at $y =$ ____.

(5) Finally, we put all these pieces together to obtain a sketch of the graph of the function $f(x)$:

Exercise 4.4. Repeat the previous exercise for the function $f(x) = \dfrac{x^2+7x+12}{x^2-16}$.