

**Exercise 3.2.4.** Solve the initial boundary value problem:

- $u(t, x)$  defined for  $t \geq 0$  and  $x \in [-1, 1]$

- $u$  satisfies

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

- $u$  satisfies the boundary conditions

$$u(t, 1) = 0 \quad u(t, -1) = 0$$

for all times  $t$ .

- $u$  satisfies the initial conditions

$$u(0, x) = 1 - x^2$$
$$\frac{\partial u}{\partial t}(0, x) = x - x^3.$$

First find a Fourier series for the solution  $u(t, x)$ . Then use Mathematica to construct some graphical representations of your solution.