TOPIC 2

Review of Calculus I

Main ideas.

- Differentiation
- Anti-differentiation
- Second fundamental theorem of calculus

Exercises.

Exercise 2.1. Compute the derivative of the following functions.

(1) $A(r) = \pi r^2$
(2) $V(r) = \frac{4}{3}\pi r^3$
(3) $S(r) = 4\pi r^2$
(4) $C(r) = 2\pi r$
(5) $f(x) = \frac{3x^2-7x}{\sqrt{x}}$
(6) $h(x) = \ln(1+x)$
(7) $q(x) = 17^x$
(8) $g(x) = \frac{\sin x}{x}$
(9) $k(t) = e^{-\frac{1}{2}t^2}$
(10) $r(x) = (1+\frac{1}{2}x)^n$, where n is a positive integer

Exercise 2.2. For each function in Exercise 2.1, find the equation of the line tangent to the graph of the function at $x = 0$, $t = 0$, $r = 1$ as appropriate. Illustrate by drawing the graph of the function and the line on the same set of axes.

Exercise 2.3. For each function in Exercise 2.1, try to find a formula for an anti-derivative. (If you cannot, explain 'what goes wrong.')

Exercise 2.4. For the functions of r in Exercise 2.1, compute the cumulative effect from $r = 0$ to $r = 1$. How might one geometrically interpret each cumulative effect?

Exercise 2.5. Consider now the function $P(t)$ from Exercise 1.2.

(1) Interpret $P(45) - P(0)$ as the cumulative effect of some function $v(t)$. What is the function v? Provide both the formula for it and also an interpretation.
(2) What is the value of the cumulative effect of v? Compare this to the distance Simeone walked. What's going on here?

Exercise 2.6. Consider now the function $d(t)$ from Exercise 1.1. What is the cumulative effect of this function (over the physically reasonable domain)? How should one interpret this effect?

[Hint: There's a lesson to be learned here – be careful about what, precisely, you are computing. How meaningful is it?]

Exercise 2.7. Define the hyperbolic cosine and hyperbolic sine functions by

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

(1) Make plots of the two functions.
(2) Show that $\cosh x$ is even, whilst $\sinh x$ is odd.
(3) Show that for any value of t, the point $(x, y) = (\cosh t, \sinh t)$ lies on the 'standard' hyperbola $x^2 - y^2 = 1$, thus justifying the descriptor 'hyperbolic.'
(4) Show that $\cosh'(x) = \sinh x$ and vice-versa.
(5) Find the equation of the line tangent to the graph of $\sinh x$ at $x = 0$.
(6) Make a table comparing the properties of the hyperbolic and 'usual' trigonometric functions. What other properties of the 'usual' trig functions do you know, which might have hyperbolic analogues?