TOPIC 12

Integration Techniques II

Exercise 12.1. Complete the following table of "basic anti-derivatives"

- $\displaystyle\int \frac{1}{v^2+1}\,dv =$
- $\displaystyle\int \frac{1}{v^2-1}\,dv =$
- $\displaystyle\int \frac{1}{v^2}\,dv =$
- $\displaystyle\int \frac{1}{\sqrt{v^2+1}}\,dv = \ln|v+\sqrt{v^2+1}|$
- $\displaystyle\int \frac{1}{\sqrt{v^2-1}}\,dv =$
- $\displaystyle\int \frac{1}{\sqrt{1-v^2}}\,dv =$

Exercise 12.2. Compute the following anti-derivatives

(1) $\displaystyle\int \frac{1}{4x^2+24x+32}\,dx$ (2) $\displaystyle\int \frac{1}{4x^2+24x+36}\,dx$ (3) $\displaystyle\int \frac{1}{4x^2+24x+40}\,dx$

Exercise 12.3. Compute the following anti-derivatives

(1) $\displaystyle\int \cos(\sqrt{x})\,dx$ (2) $\displaystyle\int \sin(\ln x)\,dx$ (3) $\displaystyle\int \tan^{-1} x\,dx$

Exercise 12.4.

(1) Show that the area of a circle of radius 3 is given by $A = 2\displaystyle\int_{-3}^{3} \sqrt{9-x^2}\,dx$.

(2) Re-write using $\sqrt{9-x^2} = 3\sqrt{1-\left(\frac{x}{3}\right)^2}$.

(3) Introduce a change of variables to reduce to an integral $A = (\#)\displaystyle\int_{?}^{?} \sqrt{1-v^2}\,dv$.

(4) Use the method of the previous homework to find the anti-derivative: Integration by parts, then add/subtract, and then wrap-around. Conclude $A = 9\pi$.

(5) Show that the area of a circle of radius r is πr^2.

Exercise 12.5. Show that the same anti-derivative strategy used for the circle (integration by parts, followed by add/subtract, and then wrap-around) also works for the anti-derivatives

- $\displaystyle\int \sqrt{x^2+1}\,dx$
- $\displaystyle\int \sqrt{x^2-1}\,dx$
- $\displaystyle\int \sqrt{x^2+9}\,dx$