TOPIC 11

Integration Techniques I

Main ideas.

- Basic anti-derivatives
- Integration by parts
- Application to area and center of area

Exercise 11.1.

(1) Compute the derivative $\dfrac{d}{dx}\left[\ln\left(x+\sqrt{x^2+1}\right)\right]$ and fully simplify. Use the result to complete the following:

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \boxed{}$$

(2) Compute the derivative $\dfrac{d}{dx}\left[\ln\left(x+\sqrt{x^2-1}\right)\right]$ and fully simplify. What anti-derivative do you now know?

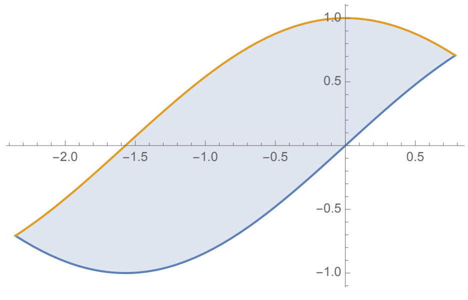
Exercise 11.2. Use partial fractions to write $\dfrac{1}{x^2-1}$ as the sum of two fractions. Use this to compute $\displaystyle\int_{-1/2}^{1/2} \frac{1}{x^2-1}\,dx$.

Exercise 11.3. Find the anti-derivatives $\displaystyle\int xe^{2x}\,dx$ and $\int x^2e^{2x}\,dx$.

Exercise 11.4. Compute the definite integral $\displaystyle\int_0^{\pi} x\cos x\,dx$.

Exercise 11.5. Compute the definite integral $\displaystyle\int_0^{\pi} e^x\sin x\,dx$

Exercise 11.6. Find the area of the region bounded by the graphs of the sine and cosine functions as shown here:



Then find x coordinate of the center of the region.

Exercise 11.7. Compute $\lim_{n\to\infty} \int_0^n e^{-x}\,dx$. Interpret the result in terms of area. Then find the x coordinate of the center of the region.

Exercise 11.8. In this problem you compute the area of a circle of radius 1.

(1) Show that the area of the circle can be represented by the integral

$$A = 2\int_{-1}^{1} \sqrt{1-x^2}\,dx.$$

(2) Use integration by parts to conclude that

$$\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}\,dx$$

(3) Algebraically rearrange this in to the form

$$\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} - \int \frac{1-x^2}{\sqrt{1-x^2}}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx$$

and conclude that

$$2\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$

(4) Return to the definite integral and conclude that the area is π.