

TOPIC 5

First steps in summation

Main ideas.

- Summation notation
- Geometric sums
- Telescoping sums

Exercises.

Exercise 5.1. Write the following sums in summation notation:

- (1) $1 + 4 + 9 + 16 + \cdots + 121$
- (2) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{15}{16}$
- (3) $1 + 3 + 9 + 27 + \cdots + 243$
- (4) $3 + 6 + 12 + 24 + \cdots + 96$
- (5) $1 - 2 + 3 - 4 + \cdots - 18$
- (6) $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{27}$
- (7) $\frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625}$
- (8) $\frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \frac{96}{625}$

Exercise 5.2. Find formulas / expressions for the following sums:

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|---------------------------|--|---|
| (1) $\sum_{k=1}^{17} k^2$ | (5) $\sum_{n=1}^{25} \frac{1}{n(n+1)}$ | (9) $\sum_{k=0}^7 \frac{1}{3^k}$ |
| (2) $\sum_{k=1}^N k^2$ | (6) $\sum_{n=1}^N \frac{1}{n(n+2)}$ | (10) $\sum_{k=0}^N \frac{2^k}{3^k}$ |
| (3) $\sum_{k=3}^{12} k^3$ | (7) $\sum_{n=2}^{42} \frac{1}{n^2-1}$ | (11) $\sum_{k=0}^{12} (-1)^k \frac{2^k}{3^k}$ |
| (4) $\sum_{k=3}^N k^3$ | (8) $\sum_{n=2}^N \frac{1}{n^2-1}$ | (12) $\sum_{k=0}^n \frac{5^k}{3^k}$ |