

MATH 132: IN-CLASS ACTIVITY

16 SEPTEMBER 2014

Get to know a classmate: Introduce yourself to two other students in the class.

- (1) What are their names?
- (2) What other classes are they taking?
- (3) Do they prefer water with or without bubbles?

Exercise 4.3:

- (1) Suppose we define the sequence S_n by $S_n = 1 + 2 + 3 + \cdots + n$. Do you think S_n converges as $n \rightarrow \infty$? Explain.
- (2) Suppose now that we define a new sequence $T_n = \frac{1}{n} + \frac{2}{n} + \cdots + \frac{n}{n}$. Do you think T_n converges as $n \rightarrow \infty$? Explain.
- (3) Next, define R_n by $R_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n}{n^2}$. Do you think R_n converges as $n \rightarrow \infty$? Explain.
- (4) Show (using Gauß' trick, listed below) that $S_n = \frac{n(n+1)}{2}$. Use this to “check” your previous responses.

Exercise 4.5: Suppose we have a sequence a_n such that $a_n \rightarrow 0$ as $n \rightarrow \infty$.

- (1) Describe the behavior of the sequence $b_n = \frac{1}{a_n}$ as $n \rightarrow \infty$? Explain your reasoning.
- (2) Describe the behavior of the sequence $b_n = \sin(a_n)$ as $n \rightarrow \infty$? Explain your reasoning.
- (3) Describe the behavior of the sequence $b_n = e^{a_n}$ as $n \rightarrow \infty$? Explain your reasoning.
- (4) Describe the behavior of the sequence $b_n = \frac{a_n}{3 + a_n}$ as $n \rightarrow \infty$? Explain your reasoning.

Gauß' trick: According to legend, the following trick was used by Euler to frustrate his math teacher. First, write the sum in two different orders

$$S_n = 1 + 2 + \cdots + (n-1) + n$$

$$S_n = n + (n-1) + \cdots + 2 + 1.$$

Then add them together

$$S_n + S_n = \underbrace{(n+1) + (n+1) + \cdots + (n+1) + (n+1)}_{n \text{ times}}$$

which can be written

$$2S_n = n(n+1).$$

The formula above follows directly from this!